A New Way to Teach, Learn, and Practice Mechanics [0]

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Software packages that enable one to use a computer to perform mathematical tasks in symbolic form have been in existence for a number of years, but have been used very sparingly to date by teachers, students, and practitioners of mechanics. For more than a decade, we have been employing one such software package, AUTOLEV, created specifically to deal with mechanics problems, and have found this to be an exceptionally effective tool in teaching mechanics at both the elementary and advanced level, as well as for performing dynamic analyses for professional purposes. It is the purpose of this article to focus attention on the benefits to be derived from using a suitable symbol manipulation program to teach, learn, and practice mechanics.

For readers not familiar with at least one of the commercially available symbol manipulation computer programs, a few examples that show what can be done with such a program may be helpful. We shall create such examples with AUTOLEV, but other symbol manipulation programs could be used.

Suppose one wished to expand the expression $(1+t)(2-3t^2+t^3)(3-4t+6t^2)$.

After invoking AUTOLEV, one would type on line (1)

```
EXPAND((1+t)*(2-3*t^2+t^3)*(3-4*t+6*t^4))
```

and then press the *Enter* key, which would cause the following to appear on the screen:

```
Result = 6 + 6*T^3 + 6*T^8 + 8*T^5 + 23*T^4 - 2*T - 18*T^6 - 17*T^2 - 12*T
```

If a 3 x 3 matrix M is given by

$$M = \begin{bmatrix} A-B & C & C-B \\ B & C+A & A \\ C & A+B & B \end{bmatrix}$$

one can obtain the inverse of M by entering the two lines

```
M=[A-B,C,C-B;B,C+A,A;C,A+B,B]
```

and

```
M_INV=INVERSE(M)
```

This elicits the responses

```
M_INV[1,1] = -(A*(A+B)-B*(A+C))/(C*(A*C+(A+C)*(B-C))-B*(B*C-(A+C)*(A
                   -B))-(A+B)*(A*(A-B)+B*(B-C)))

M_INV[1,2] = -(B*C+(A+B)*(B-C))/(C*(A*C+(A+C)*(B-C))-B*(B*C-(A+C)*(A
                   -B))-(A+B)*(A*(A-B)+B*(B-C)))
```

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etc.

As an example of a kind frequently encountered in mechanics, let **U**, **V**, and **W** be vectors defined in terms of mutually perpendicular unit vectors $\mathbf{E}_i$ (i = 1, 2, 3) as

$$\mathbf{U} = 3A\mathbf{E}_1 - 4B\mathbf{E}_2, \quad \mathbf{V} = -\mathbf{E}_2 + 2C\mathbf{E}_3, \quad \mathbf{W} = 4\mathbf{E}_1 - D\mathbf{E}_2 + A\mathbf{E}_3$$

where A, B, C, and D are constants; and consider the determination of the product P defined as

$$P = \mathbf{V} \cdot \mathbf{W} \times (\mathbf{U} \times \mathbf{V})$$

In AUTOLEV, the symbol > is used to distinguish vectors from scalars. For example, the vectors **U** and $\mathbf{E}_1$ are entered as U> and E1>. Employing this notation, all one needs to do to find P is to type the lines

```
U> = 3*A*E1> - 4*B*E2>
V> = -E2> + 2*C*E3>
W> = 4*E1> - D*E2> + A*E3>
P = DOT(V>,(CROSS(W>,CROSS(U>,V>))))
```

and then press *Enter*, thus eliciting the response

```
P = 8*A*(-1.5+B*C) - 16*C^2*(3*A+B*D)
```

That considerable amounts of labor and time are saved by using these means to find P will become clear to anyone who attempts to evaluate P by hand.

Turning to the use of symbol manipulation computer programs for pedagogical purposes, let us postulate the following situation. The students in an elementary mechanics class, having already learned how to perform basic operations of vector algebra (addition of vectors, multiplication of a vector by a scalar, etc.), are to be instructed in the determination of unknown forces by an appeal to the principle that the sum of the forces acting on a particle vanishes when the particle is at rest. As a case in point, tensile forces in three wires supporting a microphone at a height H above a horizontal floor are to be determined for various values of H, the wires being attached to vertical posts as shown in Fig. 1, this figure being displayed on a screen at the front of the classroom by means of an overhead projector. Both the instructor and each student has a computer on which AUTOLEV has been installed, and whatever appears on the instructor's computer screen is displayed on a second screen at the front of the classroom.

After stating the problem, and telling the students that F1>, F2>, F3> in Fig. 1 are the forces exerted on the microphone by the wires, while F4> is the gravitational force acting on the microphone, the instructor invokes AUTOLEV on his computer, tells the students to do so on their computers, and then begins to make entries, explaining the purpose of each entry as he makes it. The students make the same entries, a task similar to taking notes, and can ask questions and/or suggest alternative entries as the lecture proceeds.

For the problem at hand, the first two entries might be the declarations

```
FRAMES E
CONSTANTS H,T1,T2,T3
```

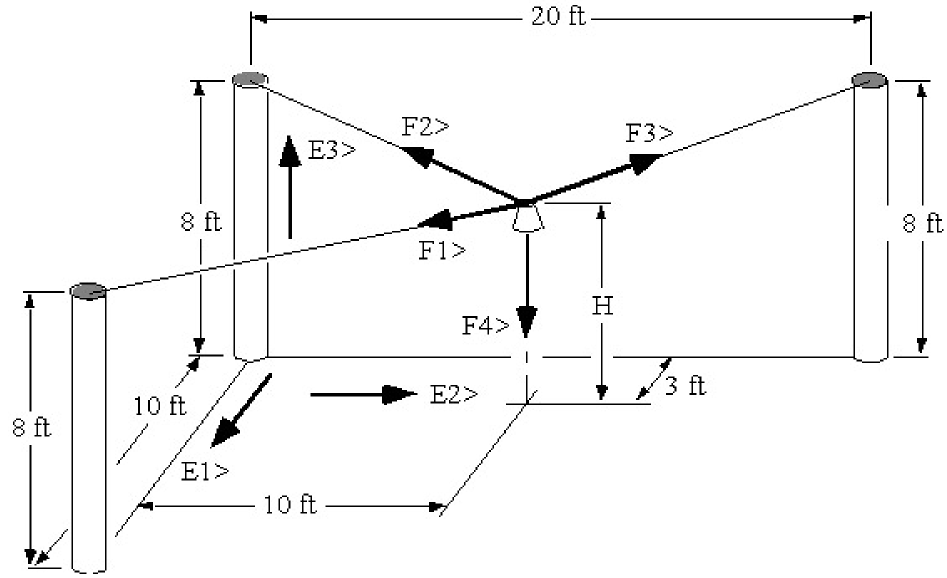

Figure 1:

accompanied by the explanation that FRAMES E serves to create the unit vectors E1>, E2>, E3> shown in Fig. 1, while T1, T2, T3 are the tensions to be determined. Next, after telling the class that each of the forces F1>, ..., F4> shown in Fig. 1 is to be expressed as the product of a unit vector and a scalar, the instructor can enter the line

```
U1> = UNITVEC(7*E1> - 10*E2> + (8-H)*E3>)
```

in order to create a unit vector having the same direction as the position vector from the microphone to the top of the post to which the wire exerting the force F1> is attached. This causes the response

```
U1> = 7/(149+(8-H)^2)^0.5*E1> - 10/(149+(8-H)^2)^0.5*E2> + (8-H)/(149+(8
-H)^2)^0.5*E3>
```

to appear on the instructor's screen, and each student now can compare this line with the corresponding one on his own screen. Once again, questions can be asked and answered, and each student can make whatever corrections are necessary to bring about agreement between his own and the instructor's expression for U1>.

The students now can be asked to create on their computers unit vectors U2> and U3> to deal with F2> and F3>, while the instructor defers making such entries until the students have had adequate time to do so.

As regards inputs made by the instructor, it is important to mention that the instructor can make these *without doing any actual typing during classroom instruction*. Instead, prior to meeting a class, the instructor can prepare (and check out) a complete AUTOLEV input file. When he next faces his class, he can display this file, as well as the responses it produces, one line at a time, simply by pressing the *Enter* key each time he wishes to show the students what the next input should be. This capability is a great boon for the instructor, for it guarantees the opportunity to present in a smooth manner well thought out, error-free, problem solutions. Moreover, use of this approach does not preclude spontaneity, for, by pressing the *Escape* key, one can go into interactive mode, type any number of commands on the spot, examine and discuss the responses to the commands, and then issue the command RUN in order to resume one line at a time presentation of the file prepared earlier.

Once the unit vectors U1>, U2>, and U3> have been created, an expression for the resultant of the forces F1>, ..., F4> is constructed with the entry

```
RESULTANT> = T1*U1> + T2*U2> +T3*U3> - 2*E3>
```

Here it is worth noting that the use of a symbol manipulator allows one to assign to mathematical entities names with high information content (such as RESULTANT>). This enhances the intelligibility of complex analyses, and facilitates communications between students and instructors.

Successive dot-multiplication of RESULTANT> with E1>, E2>, and E3> leads to three expressions that are linear in T1, T2, and T3, and that must vanish because, in accordance with the principle stated earlier, RESULTANT> is equal to 0>. Therefore, to create expressions for T1, T2, and T3, all one needs to do is to issue the command

```
SOLVE(DOT(RESULTANT>,[E1>;E2>;E3>],T1,T2,T3))
```

which yields

```
T1 = 0.6*(149+(8-H)^2)^0.5/(8-H)
T2 = 0.4*(109+(8-H)^2)^0.5/(8-H)
T3 = (109+(8-H)^2)^0.5/(8-H)
```

Finally, numerical values of T1, T2, and T3 corresponding to a particular value of H, say H = 7.95 ft, are reported with four significant digits in response to the two inputs

```
DIGITS 4
```

and

```
NUMERICAL_VALUES=EVALUATE([T1;T2;T3],H=7.95)
```

which yield

```
NUMERICAL_VALUES = [146.5; 83.52; 208.8]
```

As is now evident, the present way of solving the problem differs noticeably from the conventional one. The only manual task a student had to perform was to type the lines

```
FRAMES E
CONSTANTS H,T1,T2,T3
U1>=UNITVEC(7*E1>-10*E2>+(8-H)*E3>)
U2>=UNITVEC(-3*E1>-10*E2>+(8-H)*E3>)
U3>=UNITVEC(-3*E1>+10*E2>+(8-H)*E3>)
RESULTANT>=T1*U1>+T2*U2>+T3*U3>-2*E3>
SOLVE(DOT(RESULTANT>,[E1>;E2>;E3>]),T1,T2,T3)
DIGITS 4
NUMERICAL_VALUES=EVALUATE([T1;T2;T3],H=7.95)
```

This consumes minimal time and effort, and leaves students free to ask questions, listen to explanations, and explore alternatives. Furthermore, each student comes into possession of an easily readable record of the solution process, a record that reveals the underlying logic fully, but is free of relatively uninformative intermediate results. Even the latter, however, can be reviewed easily. All one has to do is to execute the input file and then save the file of both inputs and responses created during execution.

For teachers, use of the methodology here proposed offers not only benefits associated with classroom presentation, as has already been mentioned, but greatly facilitates the grading of homework and examinations. The burden of having to decipher poor handwriting is eliminated; solutions are free of undefined symbols because all symbols must be declared prior to being used; and logical flaws can be detected relatively easily because statements are made one line at a time, rather than all over a piece of paper, as is frequently the case in hand solutions.

In principle, the concept here set forth can be implemented to various extents and in various ways. For example, an instructor using a conventional textbook, could make available to his students one of the commercially available symbol manipulation programs and then leave it to the students to learn how to use the program and to employ it for the solution of homework problems. In our experience, this approach has not been very successful because it imposes too much of a burden on most students. A somewhat more effective way to proceed is to follow a conventional textbook, but to incorporate computerized symbol manipulation, again with a commercially available program, in the presentation of the material in the book. This can work quite well at the most elementary level, where the analytical tasks to be performed are relatively simple. The most straightforward and, we believe, most effective way is to use a textbook written specifically for this purpose and based on a symbol manipulation program specifically intended for mechanics. We have written two such books, *ENGINEERING MECHANICS ONLINE: STATICS and ENGINEERING MECHANICS ONLINE: DYNAMICS*. Individuals interested in exploring the possibility of using these can contact us by e-mail at TKANE@leland.stanford.edu.

So far, we have said nothing about the practice of mechanics at the professional level, a subject worthy of consideration separate from those of the teaching and learning of mechanics. This is where computerized symbol manipulation, in general, and AUTOLEV, in particular, can make an enormous impact, for it enables properly trained individuals to perform efficient analyses of complicated systems quickly and reliably while remaining in full control of modeling and solution processes. In this sense, teaching, learning, and practicing mechanics are intimately related to each other.